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Individual Forecasts
From Panel Data**

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EUROPEAN UNIVERSITY INSTITUTE, FLORENCE

ECONOMICS DEPARTMENT

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Printed in Italy in February 1993
European University Institute
Badia Fiesolana
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*** Optimally Combining Individual Forecasts From Panel Data**

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October 1992

Abstract

This paper explores the possibility of optimally combining individual forecasts from a small panel data set using the information contained in the error structure of past predictions. A combination schemes emerges that does not only better than almost all contemporaneous individual forecasts and the contemporaneous mean forecast, but also on average than a set of more recent predictions. Such a comparison was possible because the data used were published monthly, but predicted a quarterly variable.

*I would like to thank Prof. Robert Waldmann for many helpful comments and Prof. Agustin Maravall who got me started with the data used in this paper.

1 Introduction

Different people know different things. They also have different beliefs and might thus evaluate the same piece of new information in a different way. Consequently, an optimal combination of individual forecasts – thereby pooling the differential information – is likely to perform better than most (or any) of the underlying individual forecasts. Relying implicitly on this argument, many published forecast services actually report only some summary measure of individual forecasts such as the mean prediction¹. However, the mean is only one and generally not the optimal way of combining forecasts.

The standard technique for optimal forecast combination has been developed in papers by Bates/Granger [1969] and Granger/Ramanathan [1984]. They use the coefficients of the regression of past outcome on the different past predictions as weights for the individual forecasts. While this approach is valid to combine prediction from carefully constructed Box/Jenkins models, for example, it poses problems with panel data, first, because the panel might be unbalanced, and second, because forecasts in a panel of experts are likely to be correlated. This correlation contains actually useful information.

This paper takes consequently an alternative route and explicitly exploits the information contained in the error structure of past forecast rounds. Two different specifications of the model are tried. The preferred technique does not only fare better than almost all individual forecasters in the same forecast period, but also arrives in the upper half of the forecast performance scale when compared to more recent individual predictions for the same target period. Using the empirical error distribution it is shown that the combined forecast on average does better than any of the individual forecasts picked at random.

The paper is organized as follows: Section 2 shows that optimal weights for the combination of forecasts depend on the past error structure. Section 3 discusses different ways to estimate the error covariance matrix from the available data. Section 4 describes the empirical implementation and reports the results. Section 5 discusses a measure of evaluating the combined forecast. Section 6 summarizes.

¹One example is the monthly forecast of key monetary and financial variables published in the German weekly *Die Zeit*. When preparing these forecasts, the Munich based consulting firm *Südprojekt* combines input provided by major German banks. The idea of this paper originated from the author's attempt to obtain *Südprojekt's* data, and their request for an optimal weighing scheme in return. A second example of a survey that only publishes summary measures is the monthly *The Economist* consensus forecast.

2 Optimal Weights

Assume that the vector of individual errors \mathbf{e} is multivariate normally distributed with a mean vector \mathbf{b} and variance-covariance matrix Ω . Optimal weights for the combined forecast F^* are found by minimizing the following quadratic expectational loss function with respect to the weights:

$$\text{Min}_{\mathbf{w}} \quad E \left((Y - F^*)^2 \right) \quad (1)$$

$$\text{s.t.} \quad \mathbf{w}'\mathbf{a} = 1 \quad (2)$$

where Y is the realization of the predicted variable, \mathbf{w} is a $N \times 1$ -vector of weights and \mathbf{a} is a vector of 1's. The optimal forecast F^* is the sum of the weighted individual forecasts, each adjusted for the possible individual mean error b_i :

$$F^* = \mathbf{w}'(\mathbf{F} + \mathbf{b}) \quad (3)$$

where \mathbf{F} is the $N \times 1$ -vector of the individual forecasts. Substituting for F^* in 3, using the fact that $\mathbf{w}'\mathbf{a} = 1$, and taking expectations gives:

$$\begin{aligned} \text{Min}_{\mathbf{w}} \quad E \left((Y - F^*)^2 \right) &= \text{Min}_{\mathbf{w}} \quad E \left((Y - \mathbf{w}'(\mathbf{F} + \mathbf{b}))^2 \right) \\ &= \text{Min}_{\mathbf{w}} \quad E \left((\mathbf{w}'\mathbf{a}Y - \mathbf{w}'(\mathbf{F} + \mathbf{b}))^2 \right) \\ &= \text{Min}_{\mathbf{w}} \quad \mathbf{w}'E \left((\mathbf{a}Y - (\mathbf{F} + \mathbf{b}))^2 \right) \mathbf{w} \\ &= \text{Min}_{\mathbf{w}} \quad \mathbf{w}'\Omega\mathbf{w} \end{aligned} \quad (4)$$

That is, our loss function is a function of the error variance-covariance matrix Ω as asserted above. To minimize the expected loss, we set up the Lagrangian and take the derivative with respect to the weights \mathbf{w} :

$$L = \mathbf{w}'\Omega\mathbf{w} - \lambda(\mathbf{w}'\mathbf{a} - 1) \quad (5)$$

$$\Rightarrow \frac{\partial L}{\partial \mathbf{w}'} = 2\Omega\mathbf{w} - \lambda\mathbf{a} = 0 \quad (6)$$

$$\Leftrightarrow \Omega\mathbf{w} = \frac{\lambda}{2}\mathbf{a} \quad (7)$$

Premultiplying expression (7) by $\mathbf{a}'\Omega^{-1}$ gives:

$$\mathbf{a}'\mathbf{w} = \frac{\lambda}{2}\mathbf{a}'\Omega^{-1}\mathbf{a} = 1 \quad (8)$$

from the constraint in the optimization problem. Thus:

$$\Rightarrow \frac{\lambda}{2} = (\mathbf{a}'\Omega^{-1}\mathbf{a})^{-1} \mathbf{a} \quad (9)$$

Plugging back expressions (9) back into first-order condition (7) gives:

$$\Omega\mathbf{w} = (\mathbf{a}'\Omega^{-1}\mathbf{a})^{-1} \mathbf{a} \quad (10)$$

Premultiplying by Ω^{-1} eventually yields the optimal weights:

$$\mathbf{w}^* = (\mathbf{a}'\Omega^{-1}\mathbf{a})^{-1} \Omega^{-1}\mathbf{a} \quad (11)$$

That is, the optimal weight for forecast i is the i^{th} row sum over all elements of the inverse of the error covariance matrix Ω divided by the total sum of all elements of the inverse of the error covariance matrix:

$$w_i^* = \frac{\sum_j (\Omega^{-1})_{i,j}}{\sum_i \sum_j (\Omega^{-1})_{i,j}} \quad (12)$$

where $(\Omega^{-1})_{i,j}$ is a typical element of Ω .

The only restriction on these weights is to sum up to one. Single weights can actually take negative values which makes the range that the combined forecast can reach more flexible. Such negative weights have a distinct meaning. Assume that the true error variance/covariance matrix is known to take the following values:

$$\Omega = \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix}$$

That implies:

$$\rho = 0.8$$

$$\frac{SE_2}{SE_1} = 2.5$$

The inverse of Ω is:

$$\Omega^{-1} = \begin{pmatrix} 3 & -1 \\ -1 & 0.5 \end{pmatrix}$$

Which implies weights:

$$\begin{aligned} w_1 &= 1.33 \\ w_2 &= -0.33 \end{aligned}$$

Agnew [1985] has suggested to interpret negative weights to involve correction factors. A positive correlation as in this example means that both forecasters are likely to be either too low or too high at the same time. The less accurate forecaster's prediction, however, will probably be further away from the truth. The combination scheme in this example consequently puts a weight of one on the forecast more likely to be accurate adding a correction factor equal to one third of the difference between the two forecasts. That is, if e.g.:

$$\begin{aligned} F_1 &= 6 \\ F_2 &= 8 \end{aligned}$$

Then:

$$\begin{aligned} F^* &= (1.33) \times 6 + (-0.33) \times 8 \\ &= 5.34 \end{aligned}$$

The difference between the low variance forecast and the combined forecast is $F_1 - F^* = 0.66 = 0.33 \times (F_2 - F_1)$, one third the difference between the two forecasts as asserted above.

3 Estimating Omega

As has been shown in section 2, the optimal weights for combining individual forecasts depend on the error covariance structure of past forecast rounds. This matrix Ω has been developed in its theoretical form. There are $N \times (N - 1)/2$ distinct elements in the theoretical Ω matrix – too many to be estimated from the data set at hand. The task is consequently to find a suitable empirical estimator which is, on one hand, flexible

enough to capture the information in the past error structure, but on the other hand, does not exhaust the degrees of freedom. This section suggests two suitable estimators.

A first point to note, however, is that the simple average, as reported in some surveys, would only be the optimal combination if the covariance between all pairs (i, j) as well as the own variances of forecasts errors for all i were the same. This is not likely to be the case. A second, precision-weighted, combination would use the inverse of the own variances as weights. This improves upon the simple average because it allows for individual variation, but still neglects the covariances terms.

One natural choice as estimator for Ω , in the following called the Rho model, is the covariance matrix developed in an earlier paper for the tests of rationality in survey expectation data. The null hypothesis of Rational Expectations postulates serially uncorrelated own forecast errors. However, across agents forecasts error are likely to be correlated for the same period since the agents are surprised by the same aggregate shock hitting the economy. This implication for the error structure has been formalized in the following assumptions:

$$E(\varepsilon_{i,t}^2) = \sigma_i^2, \quad \text{for all } t=1 \dots T; i=1 \dots N \quad (13)$$

$$E(\varepsilon_{i,t}\varepsilon_{j,t}) = \rho\sigma_i\sigma_j, \quad \text{for all } t \text{ and } i \neq j \quad (14)$$

where $\varepsilon_{i,t}$ is forecaster i 's prediction error in period t , σ_i is the standard deviation of i 's prediction error, and ρ is a correlation coefficient. This specification allows for heteroscedasticity of the disturbances across units and for non-zero contemporaneous correlation between the disturbances in different units. It reduces the number of parameters to be estimated to $N+1$. The common correlation coefficient ρ reflects the assumption of an aggregate shock to the economy. The resulting, theoretical $(N \times N)$ -covariance matrix under the null hypothesis of Rational Expectations is then:

$$\Gamma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \dots & \rho\sigma_1\sigma_N \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & \dots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \rho\sigma_N\sigma_1 & \dots & \dots & \sigma_N^2 \end{pmatrix} \quad (15)$$

For estimation, error series for all individual forecasters can be calculated as the difference between outcome minus individual forecasts:

$$e_{t,i} = Y_t - F_{t,i} \quad (16)$$

These series can then be used to estimate the elements of Γ to obtain a first estimate of the error covariance matrix Ω . The estimators for σ_i and ρ resp. are:

$$\hat{\sigma}_i = \sqrt{\frac{\mathbf{e}_i' \mathbf{e}_i}{T}} \quad (17)$$

$$\hat{\rho} = \frac{\sum_{i \neq j} \rho_{i,j} * (N_{i,j} - 1)}{\sum_{i \neq j} (N_{i,j} - 1)} \quad (18)$$

where \mathbf{e}_i is the vector of individual errors for agent i , $\rho_{i,j}$ is the correlation between any pair of errors i and j , and $N_{i,j}$ is the number of observations with non-missing forecasts for both participants i and j . That is, the common correlation coefficient is calculated as a weighted average of all pairwise correlation coefficients.

In the empirical implementation, optimal weights would then be found according to expression (11), replacing the inverse of the theoretical error covariance matrix Ω by its estimate $\hat{\Omega}^{-1}$. Individual biases could be estimated as mean error $\sum_t \frac{Y_{i,t} - F_{i,t}}{T}$ which is equivalent to running OLS regressions of the forecast error series on a constant. The number of parameters to be estimated with bias adjustment is $2N+1$, without $N+1$.

A second estimator for the unknown error covariance matrix Ω has been suggested by Figlewski [1983]. It comes from the finance literature that seeks to determine the optimal portfolio choice for holding risky assets. The technical problem is similar to the one encountered here as there are typically too many correlation terms between risky assets, $N \times (N - 1)/2$, than can be estimated. Under the assumption of a multivariate relationship between the returns on individual assets, the model centers on estimating the relationship of each individual asset with a Single Index.

Formulated in terms of forecast errors, the model starts by assuming that individual errors are composed of an individual bias, an individual constant times the single index, and an individual random term.

$$e_i = \gamma_i + \delta_i \bar{e} + \epsilon_i \quad (19)$$

where e_i is the individual error defined as outcome minus individual forecast, \bar{e} is the mean (or market) error which is taken to be the single index, and ϵ_i is an individual random error term with mean zero and variance $\sigma_{\epsilon,i}$. Taking the mean error as single index makes a lot of sense, but entails a problem in that the random error term is not independent of \bar{e} since the individual forecast is used to calculate the mean error. For the time being, this complication is ignored and we assume all the following results to hold approximately. Assuming that a relation of type (19) holds for all forecasters $i = 1 \dots N$, the model is then characterized by the following relationships:

$$\begin{aligned} E(\epsilon_i) &= 0 && \text{for all } i \text{ (by construction)} \\ E((\epsilon - 0)(\bar{e} - E(\bar{e}))) &= 0 && \text{for all } i \text{ (by assumption)} \\ E(\epsilon_i \epsilon_j) &= 0 && \text{for all } i \neq j \text{ (by assumption)} \end{aligned}$$

$$\begin{aligned} E(\epsilon_i) &= \sigma_{\epsilon,i}^2 && \text{for all } i \text{ (by definition)} \\ E(\bar{e} - E(\bar{e})) &= \nu^2 && \text{(by definition)} \end{aligned}$$

To construct the error covariance matrix Ω , we need to derive the following three results for the expected value of the individual error, its variance, and the covariance between any two errors respectively. First, the expected value of the individual error is:

$$\begin{aligned} E(e_i) &= E(\gamma_i + \delta_i \bar{e} + \epsilon_i) \\ &= \gamma_i + \delta_i E(\bar{e}) \end{aligned} \quad (20)$$

which establishes the expected value of the individual error. Second, the variance of the individual error is:

$$\begin{aligned} \sigma_{\epsilon,i}^2 &= E(e_i - E(e_i))^2 \\ &= E((\gamma_i + \delta_i \bar{e} + \epsilon_i) - (\gamma_i + \delta_i E(\bar{e})))^2 \\ &= E(\gamma_i(\bar{e} - E(\bar{e})) + \epsilon_i)^2 \\ &= \gamma_i^2 E(\bar{e} - E(\bar{e}))^2 + 2\gamma_i E(\epsilon_i(\bar{e} - E(\bar{e}))) + E(\epsilon_i)^2 \\ &= \gamma_i^2 \nu^2 + \sigma_{\epsilon,i}^2 \end{aligned} \quad (21)$$

since $E(\epsilon_i(\bar{e} - E(\bar{e})))$ is assumed to equal zero which establishes the variance of the individual error. Last, the covariance between any errors i and j is:

$$\begin{aligned} \sigma_{i,j} &= E[((\gamma_i + \delta_i \bar{e} + \epsilon_i) - (\gamma_i + \delta_i E(\bar{e}))) \times ((\gamma_j + \delta_j \bar{e} + \epsilon_j) - (\gamma_j + \delta_j E(\bar{e})))] \\ &= E[(\delta_i(\bar{e} - E(\bar{e})) + \epsilon_i) - (\delta_j(\bar{e} - E(\bar{e})) + \epsilon_j)] \\ &= \delta_i \delta_j E(\bar{e} - E(\bar{e}))^2 + \delta_j E(\epsilon_i(\bar{e} - E(\bar{e}))) + \delta_i E(\epsilon_j(\bar{e} - E(\bar{e}))) + E(\epsilon_i \epsilon_j) \\ &= \delta_i \delta_j \nu^2 \end{aligned} \quad (22)$$

since the last three terms in the second to-the-last equation are all assumed to equal zero. With these three results, the variance-covariance matrix can be constructed as follows:

$$\Omega = \begin{pmatrix} \delta_1^2 \nu^2 + \sigma_{\epsilon,1}^2 & \delta_1 \delta_2 \nu^2 & \cdots & \cdots \\ \delta_2 \delta_1 \nu^2 & \delta_2^2 \nu^2 + \sigma_{\epsilon,2}^2 & \cdots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \cdots & \cdots & \delta_n^2 \nu^2 + \sigma_{\epsilon,n}^2 \end{pmatrix} \quad (23)$$

Ignoring the fact that \bar{e} is not independent from e_i , the following regressions can be run to estimate the individual components of the variance-covariance matrix Ω :

$$e_{i,t} = g_i + d_i \bar{e}_t + u_{i,t} \quad (24)$$

Given the assumptions of the Single Index model, $cov(e_i, \bar{e}) \cong \delta_i \nu^2$, and $var(\bar{e})$ is equal to ν^2 . It follows:

$$\begin{aligned} d_i &= \frac{cov(e_i, \bar{e})}{var(\bar{e})} \\ &\cong \delta_i \end{aligned} \quad (25)$$

and:

$$\begin{aligned} g_i &= E(e_i) - \delta_i E(\bar{e}) \\ &= \gamma_i - \delta_i E(\bar{e}) \end{aligned} \quad (26)$$

The intercept term, an individual bias term, can be recovered from the bias of the expected mean error times the individual relation of the i^{th} error with the mean error. The number of parameters to be estimated with the Single Index model is $3N+1$; if the bias vector is restricted to zero, the number of parameters reduces to $2N+1$.

4 Empirical Implementation

The techniques outlined in section 3 have been applied to the panel of survey expectations used in an earlier paper (Ehrbeck [1992]). The forecasted variable is the annualized discount rate on new issues of 91-day U.S. Treasury bills, based on the weekly auction results. The publication is monthly; the forecasts are for the quarters of the calendar year. The data was consequently split into three homogeneous panels of first month, second month and third months forecasts resp. The realization data is from the U.S. Federal Reserve Bulletin.

For the empirical tests, only forecasters were included who reported regularly over the sample period from December 1984 to November 1991. The cross-section dimension of the data was $N = 27$. The time series dimension was for each of the three sets $T = 28$. I took the first 15 observations as starting history to calculate the necessary error series for the Rho model and to run the regressions for the Single Index model. Based on this past error structure, optimal weights and the resulting optimally combined forecast were

calculated for the 16th period. The procedure was then repeated 13 times, each time adding the most recent period to the history, re-estimating the error covariance matrix Ω , and finding the next period optimal forecast.

This rolling procedure was applied to the set of first month, second month, and third month forecasts for the current quarter separately. Expectations for overlapping forecast horizons would imply a more complicated error structure than the present versions of both the Rho and the Single Index model allow for. Both models were run with and without bias adjustment. The resulting series of 13 optimally combined forecasts obtained with the Rho model and the Single Index model were then compared to the simple average and the precision-weighted average of the same period forecasts.

Table 1 summarizes the performance of the six different combined forecasts based on the individual first month of the quarter predictions. As an evaluation criterion, the root mean squared error (RMSE) was used:

$$RMSE = \sqrt{\frac{\sum_t (Y_t - F_t^*)^2}{T}} \quad (27)$$

With this criterion, the Rho model without bias adjustment does best. It has a RMSE 18.42 percent lower than the simple average. The Rho model with bias adjustment does 12.32 percent better than the simple average, the precision-weighted average (Var) 10.02 percent, the Single Index with bias adjustment 5.06 percent, and the Single Index without bias adjustment 1.66 percent better.

The second part of table 1 reports the RMSEs for the individual forecasts from the same period. Note that one forecaster (no. 6) is doing better than the Rho model combination. The third part of table 1 reports for comparison purposes the RMSEs of the second month of the quarter predictions for the same target quarter. 12 forecasters do better than the Rho model prediction based on the first months forecasts, but 15 do worse.

Similarly, table 2 summarizes the relative performance of the six combined forecasts based on the individual second month of the quarter predictions. Precision-weighted average by 16.05 percent, Single Index by 5.77 percent, and Rho model by 4.12 percent do better than the simple average. However both models, Single Index and Rho, do worse with bias adjustment than without, even worse than the simple average. Note that of the individual forecasters, applying the RMSE criterion, one (no. 15) does better than the Rho model which we have chosen as benchmark. In the third part of table 2, the RMSEs of the individual forecasts made in the third month of the quarter are reported. Seven forecasters fare better than the Rho model prediction based on the second month forecasts.

Table 3 finally, reports similar results for the third month of the quarter predictions. All combinations do better than the simple average. Single Index, Rho model, and precision-weighted average do so almost by the same margin. Of the individual fore-

Table 1
Combining Individual Forecasts
Current Quarter, First Month

Model		RMSE	Rel. Performance
Average		0.177	-
SI		0.174	-1.66 %
SI _{wc}		0.168	-5.06 %
Var		0.159	-10.02 %
Rho		0.145	-18.42 %
Rho _{wc}		0.155	-12.32 %

RMSE of Individual 1st Month Forecasts for Current Quarter					
1. 0.295	2. 0.235	3. 0.218	4. 0.320		
5. 0.250	6. 0.121*	7. 0.310	8. 0.382		
9. 0.346	10. 0.285	11. 0.433	12. 0.176		
13. 0.314	14. 0.219	15. 0.174	16. 0.217		
17. 0.360	18. 0.354	19. 0.534	20. 0.192		
21. 0.262	22. 0.165	23. 0.288	24. 0.160		
25. 0.384	26. 0.205	27. 0.216			

Competing RMSE of Ind. 2nd Month Forecasts for Current Quarter					
1. 0.271	2. 0.168	3. 0.291	4. 0.066*		
5. 0.173	6. 0.053*	7. 0.182	8. 0.741		
9. 0.168	10. 0.140*	11. 0.246	12. 0.092*		
13. 0.146	14. 0.102*	15. 0.038*	16. 0.270		
17. 0.215	18. 0.147	19. 0.072*	20. 0.135*		
21. 0.210	22. 0.048*	23. 0.210	24. 0.059*		
25. 0.102*	26. 0.173	27. 0.108*			

* Lower RMSE than Rho Model

Table 1: First Month Optimal Combination

Table 2
Combining Individual Forecasts
Current Quarter, Second Month

Model		RMSE	Rel. Performance	
Average		0.049	-	
SI		0.046	-5.77 %	
SI _{wc}		0.060	21.51 %	
Var		0.041	-16.05 %	
Rho		0.048	-4.12 %	
Rho _{wc}		0.052	4.26 %	

RMSE of Individual 2nd Month Forecasts for Current Quarter

1. 0.271	2. 0.168	3. 0.291	4. 0.066
5. 0.173	6. 0.053	7. 0.182	8. 0.741
9. 0.168	10. 0.140	11. 0.246	12. 0.092
13. 0.146	14. 0.102	15. 0.038*	16. 0.270
17. 0.215	18. 0.147	19. 0.072	20. 0.135
21. 0.210	22. 0.048	23. 0.210	24. 0.059
25. 0.102	26. 0.173	27. 0.108	

Competing RMSE of Ind. 3rd Month Forecasts for Current Quarter

1. 0.249	2. 0.169	3. 0.092	4. 0.262
5. 0.143	6. 0.039*	7. 0.334	8. 0.212
9. 0.157	10. 0.104	11. 0.153	12. 0.115
13. 0.150	14. 0.086	15. 0.028*	16. 0.030*
17. 0.165	18. 0.150	19. 0.043*	20. 0.032*
21. 0.104	22. 0.140	23. 0.048	24. 0.078
25. 0.029	26. 0.080	27. 0.023	

* Lower RMSE than Rho Model

Table 2: Second Month Optimal Combination

Table 3
Combining Individual Forecasts
Current Quarter, Third Month

Model		RMSE	Rel. Performance
Average		0.033	-
SI		0.022	-34.49 %
SI _{wc}		0.027	-19.35 %
Var		0.021	-37.69 %
Rho		0.021	-37.34 %
Rho _{wc}		0.023	-31.56 %

RMSE of Individual 3rd Month Forecasts for Current Quarter

1. 0.236	2. 0.162	3. 0.088	4. 0.251
5. 0.143	6. 0.037	7. 0.322	8. 0.210
9. 0.150	10. 0.104	11. 0.153	12. 0.111
13. 0.147	14. 0.088	15. 0.026	16. 0.032
17. 0.165	18. 0.145	19. 0.052	20. 0.029
21. 0.097	22. 0.140	23. 0.048	24. 0.075
25. 0.030	26. 0.080	27. 0.026	

* Lower RMSE than Rho Model

Table 3: Third Month Optimal Combination

casters, none does better than the Rho model according to the RMSE criterion. Note that Rho and Single Index when adjusting for the bias do worse than without bias adjustment. This might be due to the fact that the estimated bias terms are not stable over time or due to the large number of parameters to be estimated which reduces the out-of-sample performance.

5 Evaluation of Combined Forecast

Results in section 3 suggest that the optimally combined forecast does not only better than most of the contemporaneous individual forecasts but also than a majority of later individual forecasts for the same target period. In this section, we propose a measure that shows that the combined forecast does on average better than any contemporaneous individual forecast for any symmetric loss function that punishes bigger mistake more than smaller ones. It is also better on average than any individual forecast from the set of more recent predictions picked at random for any symmetric convex loss function which is a less stringent criterion.

Figure 1

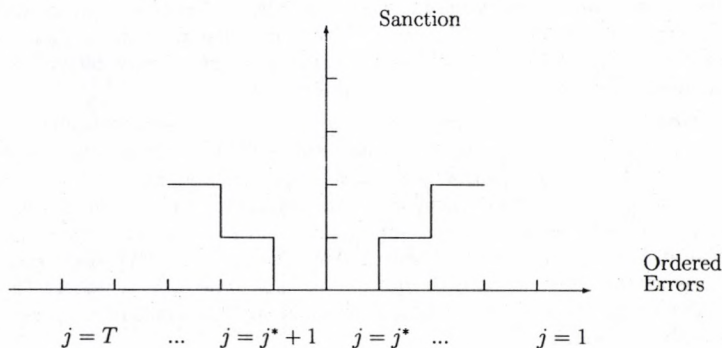


Figure 1: Step-Loss Function

For explanation of the concept consider figures 1 and 2. On the x-axis are from the right to the left the errors of the combined forecast in decreasing order: $\text{Error}_{j=1}$ is the largest positive error, $\text{error}_{j=2}$ the second largest, and so on. $\text{Error}_{j=j^*}$ is the smallest positive error, and $\text{error}_{j=T}$ the smallest negative error. On the y-axis is the sanction for

Figure 2

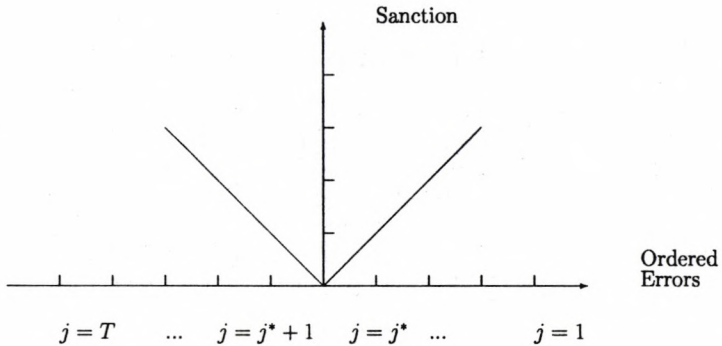


Figure 2: Symmetric Loss Function

making a forecast error. Plotting now in this graph all individual error over time and across agents gives an idea of the empirical distribution of individual errors with respect to the errors of the combined forecast. If there are more individual errors larger than error than there should be according to the rank of j , then the combined forecast does on average better than the set of individual forecasts.

Figure 1 shows an evaluation criterion for the performance of the combined forecast that only makes sure that larger mistakes will result in a larger loss. Regardless of the size of the individual error, with such a loss function we would only count the number of occurrences of individual errors larger than the largest error of the combined forecast, the number of individual errors larger than the second largest error of the combined forecast, and so on. Figure 2 shows a less demanding loss function: Here errors enter the loss function with their size multiplied by some constant. This reflects the most stringent convex loss function one could apply to evaluate the performance of the combined forecast.

The proposed measures capture the concept underlying figures 1 and 2: Measure Φ_1 calculates the average number of occurrences of individual errors larger than j th error of the combined forecasts obtained with the Rho model, weighted by the rank of the j^{th} error:

$$\Phi_1 = \frac{1}{T} \left(\sum_{j=1}^{j^*} \frac{\sum_{i=1}^N \sum_{t=1}^T N_{it} = 1}{j} \text{ if } e_{it} > e_j^{r_{ho}} \right) + \dots \quad (28)$$

$$\dots + \frac{1}{T} \left(\sum_{j=j^*+1}^T \frac{\sum_{i=1}^N \sum_{t=1}^T N_{it} = 1}{j} \text{ if } -e_{it} < -e_j^{r_{ho}} \right)$$

The combined forecast is better on average than the set of individual forecasts for any loss function if $\Phi_1 > \Psi_1$ which is in this case just the average number of individual forecasts:

$$\Psi_1 = \frac{\sum_{j=1}^J n_j}{T} \quad (29)$$

where n_j is the number of individual forecasts per period. In measure Φ_2 individual errors enter with their size multiplied by some constant; with $q=k$ this would imply a symmetric loss function:

$$\Phi_2 = \frac{1}{T} \left(\sum_{j=1}^{j^*} \frac{\sum_{i=1}^N \sum_{t=1}^T N_{it} = k * e_{it} \text{ if } e_{it} > e_j^{r_{ho}}}{j} \right) + \dots \quad (30)$$

$$\dots + \frac{1}{T} \left(\sum_{j=j^*+1}^T \frac{\sum_{i=1}^N \sum_{t=1}^T N_{it} = q * e_{it} \text{ if } -e_{it} < -e_j^{r_{ho}}}{j} \right)$$

where $k, q > 0$. Again, the combined forecast does better on average than the set of individual forecasts if the measure $\Phi_2 > \Psi_2$ which is now the average forecast error weighted by the number of individual forecasts in the period the error occurred:

$$\Psi_2 = \frac{\sum_{j=1}^{j^*} e_j * k * n_j + \sum_{j=j^*+1}^T -e_j * q * n_j}{T}$$

These measures have been calculated for all forecast series of the Rho model obtained in section 3 and compared to the set of contemporaneous forecasts and more recent forecasts. Table 4 summarizes the results. The combined forecast according to the proposed measure dominates the set of contemporaneous forecasts for any symmetric loss function, i.e. $\Phi_1 > \Psi_1$, and the set of more recent forecasts for any convex symmetric loss function, i.e. $\Phi_2 > \Psi_2$.

Table 4
Evaluating the Combined Forecast

		First Month Combination			
		Φ_1	Ψ_1	Φ_2	Ψ_2
Set of individual					
1st month forecasts		29.3	23.7	9.6	3.0
Set of individual					
2nd month forecasts		16.5	23.2	4.4	2.9
		Second Month Combination			
		Φ_1	Ψ_1	Φ_2	Ψ_2
Set of individual					
2nd month forecasts		35.4	23.2	6.5	1.0
Set of individual					
3rd month forecasts		21.9	22.5	3.5	0.9
		Third Month Combination			
		Φ_1	Ψ_1	Φ_2	Ψ_2
Set of individual					
3rd month forecasts		36.4	22.3	4.1	0.4

Table 4: Evaluating Combined Forecasts

6 Summary

This paper explores the idea of optimally combining individual forecasts from a small panel data set. The relative performance of two different specifications of a model that seeks to exploit the information contained in the past individual errors are compared with the simple average and the precision-weighted average.

Employing rolling estimation, three different sets of forecasts series from the different forecasts months were obtained. In all three cases, the combined forecasts with optimizing weights recovered from the error covariance structure fared better than the mean prediction. One of the models used was taken from the literature on testing for rationality in survey expectation data, the other from the finance literature which solves the similar problem of having to reduce the number of parameters to be estimated in an error covariance matrix with $N \times (N - 1)/2$ distinct terms. For the three runs, both model did better with the mean bias vector restricted to zero.

The combined forecasts based on past individual predictions did better than almost any of the contemporaneous individual forecasts. Using a proposed measure, it was shown that the combined forecasts from the Rho model which was chosen as a benchmark did on average also better than the set of more recent individual forecasts for any convex symmetric expectational loss function. This evaluation was possible because the data used are published monthly, but predict a quarterly variable.

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